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# DISTRIBUTIONS FOR A WIDE VARIETY OF BLUNT

## BODIES AT ZERO ANGLE OF ATTACK

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#### SUMMARY

A method is presented for determining shock envelopes and pressure distributions for a variety of blunt bodies at zero angle of attack. Correlation functions obtained from exact solutions are used to relate the shock standoff distance, at the stagnation and sonic points, to the body geometry. These functions were obtained for a perfect gas but may be applied for real gases in equilibrium flows. The method is restricted to cases where the bow shock is detached from the body and the flow over the forward face is subsonic. Results given by the method are shown to be in good agreement with experimental values.

#### INTRODUCTION

A problem currently receiving wide attention is that of predicting the shock envelope and pressure distribution over the forward face of axisymmetric blunt bodies at atmospheric entry.

Exact solutions have been presented for spheres, ellipsoids, and paraboloids by Van Dyke (ref. 1). In this work and that of subsequent investigators most exact solutions are for the indirect problem; a shock shape is prescribed and the resultant body shape is sought. Many trial solutions are usually necessary to find a body shape that approximates the one for which a solution is wanted.

An approximate "direct" method to define the shock trace at zero angle of attack is included in the analysis of reference 2. This method is restricted to spherically blunted bodies with sharp corners. Extensions of this method which permit the calculation of the entire forward shock envelope and the pressure distribution for these bodies have been published (refs. 3 and 4). The exact and approximate solutions of these references are applicable to a relatively narrow choice in body shape.

This investigation was undertaken to develop methods for predicting the shock envelopes and pressure distributions over the forward face of a wide variety of body shapes at zero angle of attack. Particular attention is given to ellipsoids (ranging from a sphere to a flat disk), spherical or

flat-faced bodies with an arbitrary degree of corner rounding, and spherically blunted, large-apex-angle cones. The method developed is restricted to those cases in which the bow shock is detached from the body and the flow over the forward face of the body is subsonic.

# NOTATION

A	coefficient in transformed pressure distribution function
а	semiminor axis of ellipsoid
$\mathbb{B}_{b}$	body bluntness parameter, $y^2 = 2R_b x_b - B_b x_b^2$ (ref. 1)
$B_s$	shock bluntness parameter, $y^2 = 2R_S x_S - B_S x_S^2$ (ref. 1)
ъ	semimajor axis of ellipsoid
C	coefficient in transformed pressure distribution function
G	shock correlation function (see eqs. (1))
M	free-stream Mach number
m; st	slope of transformed pressure distribution function at stagnation point
$m_{\stackrel{\bullet}{X}}$	slope of transformed pressure distribution function at sonic point
р	pressure on body
p <sub>st</sub>	pressure at stagnation point on body
p <sub>*</sub>	pressure at sonic point on body
$R_{b}$	body surface radius on axis of symmetry
$R_{b*}$	body surface radius at sonic point of an ellipsoid
$R_{oldsymbol{b}\Phi}$	body surface radius at point on surface inclined $\Phi$ degrees with respect to a plane normal to free-stream direction
$R_{\rm S}$	shock radius on axis of symmetry
$r_c$	corner radius of body
$\mathbf{r}_{\mathrm{m}}$	semidiameter of body
s	distance along body surface from stagnation point

distance along body surface from stagnation point to sonic point on  $s_*$ body  $s^{\dagger}/s^{\dagger}$ independent variable for transformed pressure distribution function  $s_{+}^{\dagger}/s_{\perp}^{\dagger}$ tangent point of transformed pressure distribution functions  $V_1$ free-stream velocity  $\Lambda^{r}$ velocity at sonic point on body streamwise distance from apex of body to sonic point on body  $\mathbf{x}_{\mathbf{b}}$ streamwise distance from apex of shock to point on shock at distance  $x_s$ y from axis of symmetry normal distance from axis of symmetry У normal distance from axis of symmetry to sonic point on body  $y_*$ Z coordinate axis of transformed pressure distribution function dependent variable in transformed pressure distribution function γ specific heat ratio shock standoff distance from body in free-stream direction Δ shock standoff distance from body on axis of symmetry in free-stream  $\Delta_{0}$ direction shock standoff distance from sonic point on body in free-stream  $\triangle_{\mathbf{x}}$ direction inclination on forward body surface at the tangent point with corner  $\in$ radius, r normal distance from body surface (sketch (g)) η shock surface inclination at distance, y, from axis of symmetry with θ respect to plane normal to free-stream direction shock surface inclination at a point opposite the sonic point on the  $\theta_{*}$ body with respect to a plane normal to the free-stream direction  $\theta_{*o}$ shock surface inclination, at a point opposite the sonic corner of a flat disk, with respect to a plane normal to the free-stream direction pressure correlation function

- $\rho_1$  density of free stream
- $\rho_{2}$  density of stream behind normal shock
- $ho_{\rm st.}$  density of gas at stagnation point
- σ shock layer thickness normal to body (sketch (g))
- $\Phi$  body surface inclination with respect to plane normal to free-stream direction
- $\Phi_{\mbox{\scriptsize \#}}$  body surface inclination at sonic point with respect to plane normal to free-stream direction
- $\Phi_{*1}$  body surface inclination of a sphere at the sonic point with respect to a plane normal to the free-stream direction

#### ANALYSIS

The analysis will be developed in three parts. First, general relationships and assumptions involving body and shock geometry are introduced. Next, application of the shock-body relationships to a method for calculating the shock envelopes of a variety of blunt bodies is demonstrated. The calculation involved is simplified by the use of a nomograph. Finally, a general method for estimating the pressure distribution for the bodies considered is presented.

# General Shock-Body Relationships

Shock shape.- Reference 1 defined shock traces as conic sections of suitable bluntness  $B_{\rm S}$  (see Notation) to obtain the shock solution for a range of conic section blunt bodies. In the present investigation, it was assumed that conic sections can closely approximate the shock trace for a larger variety of blunt bodies than were considered in reference 1.

Stagnation-point correlation. - A relationship involving shock standoff distance, shock shape, and normal-shock density ratio was pointed out in reference 5 and used in reference 2. A simplified form of this correlation, as used in the present investigation, is

$$G = \left(1 + \frac{\Delta_{O}}{R_{b}}\right) \frac{\Delta_{O}}{R_{S}} = \left(1 + \frac{R_{S}\Delta_{O}}{R_{b}R_{S}}\right) \frac{\Delta_{O}}{R_{S}}$$
 (la)

$$\frac{\Delta_{o}}{R_{s}} = \frac{\sqrt{1 + 4G\left(\frac{R_{s}}{R_{b}}\right) - 1}}{2\left(\frac{R_{s}}{R_{b}}\right)}$$
(1b)

G is a function of the free-stream gas and Mach number and was evaluated by substituting values for  $\Delta_{\rm O}/R_{\rm S}$  and  $\Delta_{\rm O}/R_{\rm b}$  from reference l into equation (la). Figure l shows G as a function of the reciprocal normal-shock density ratio  $\rho_1/\rho_2$  (rather than Mach number) for two values of the gas specific heat ratio,  $\gamma_{\rm *}$ 

Sonic point shock standoff correlation. The theoretical results of reference 1 and numerous experimental results for a variety of body shapes indicate that the ratio,  $\triangle_{\mathbf{x}}/\mathbf{y}_{\mathbf{x}}$ , is primarily a function of the normal-shock density ratio and secondarily of  $\gamma$  pertinent to the free-stream gas in question. Sketch (a) shows  $\triangle_{\mathbf{x}}$ , the shock thickness measured in the free-stream direction from the sonic point on the body at  $\mathbf{y}_{\mathbf{x}}$ , the vertical distance from the axis of symmetry.

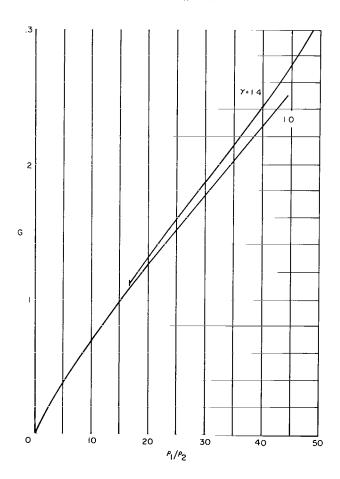
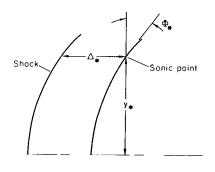


Figure 1. - The G function.

Sonic point inclination. - A solution for the infinite shock-density ratio was used to determine the inclination of the sonic point on conic-section bodies. The results were then applied to finite shock-density ratios and to bodies of other shapes.

The angle,  $\Phi_{\rm x}$ , associated with the sonic point is the angle between a normal to the free-stream direction and a line tangent to the body at the sonic point (sketch (a)). This angle was determined with the Busemann solution for an infinite shock-density ratio as a function of the body bluntness parameter,



Sketch (a)

 $\mathrm{B}_{\mathrm{b}}$  (see Notation). Details of the solution are presented in appendix A. The essential result is

$$\frac{\left[\left(B_{b}-1\right)\sin^{2}\Phi_{\star}+1\right]^{2}}{2(B_{b}-1)\sin\Phi_{\star}}\left\{\frac{1}{\sqrt{B_{b}-1}}\sin^{-1}\left[\frac{\sqrt{B_{b}-1}\sin\Phi_{\star}}{\sqrt{(B_{b}-1)\sin^{2}\Phi_{\star}+1}}\right]-\frac{\sin\Phi_{\star}}{(B_{b}-1)\sin^{2}\Phi_{\star}+1}\right\}$$

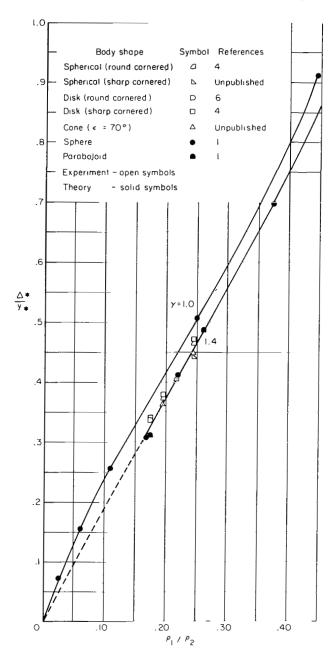


Figure 2.- Correlation curves of shock-standoff distance at sonic point,  $\Delta_*/y_*$ .

$$= (1 - \sin^2 \Phi_*) - \frac{p_*}{p_{st}}$$
 (2)

In the present application of equation (2), the gas was assumed perfect. Since the theory requires an infinite shock-density ratio, the value  $p_*/p_{st} = 0.607$ , appropriate to a gas with  $\gamma = 1.0$ , was used.

The results of equation (2) for  $\sin \Phi_*$  are presented in figure 3 normalized with respect to the value for a sphere ( $\sin \Phi_{*1}$ ). This normalized value is plotted as a function of the more tangible variable,  $a/b = B_b^{-1/2}$ , rather than as a function of  $B_b$ .

Although the absolute value for  $\sin \Phi_*$  given by equation (2) may not be correct for a finite density ratio, it is assumed that the ratio  $\sin \Phi_{\star}/\sin \Phi_{\star 1}$  will not differ greatly from that given by exact theory. 1 Thus, if the value of  $\sin \Phi_{\mathbf{x}\mathbf{1}}$  is known for a sphere in a given free stream, the value of  $\sin \Phi_{\star}$  for an ellipsoid can be determined from the curve of figure 3. Values of  $\Phi_{*1}$ , for a sphere at various free-stream conditions (ref. 1) are presented in figure 4 for convenience in such calculations.

If a blunt body does not have a conic section, the parameter  $\mbox{\ensuremath{B}}_b$  is not appropriate. However, the

The validity of figure 3 for finite shock-density ratios is substantiated to some degree by the points from reference 1 for ellipsoids of moderate values of a/b.

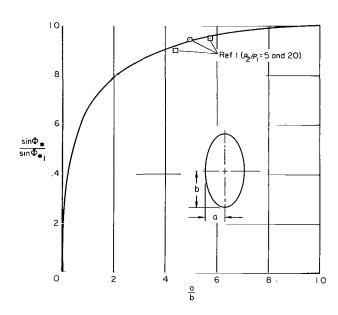


Figure 3.- Sonic angle,  $\Phi_{\mathbf{x}},$  on ellipsoids.

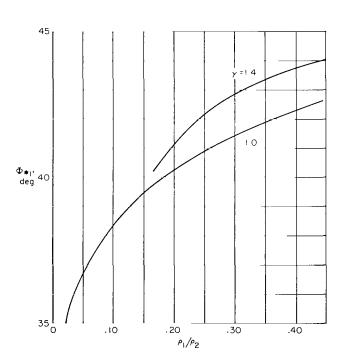


Figure 4.- Sonic angle for a sphere  $(B_b = 1, a/b = 1)$ .

sonic-point location on other types of blunt bodies may be estimated on the basis of the foregoing analysis. To this end, it is assumed that a shock in the form of a conic section is appropriate to all classes of blunt bodies considered here. example, a blunt body with a rounded corner of radius  $r_{\text{C}}$  is illustrated in sketch (b). When the flow has an infinite shock-density ratio  $(\rho_2/\rho_1)$ , the shock approaches coincidence with the body in the vicinity of the sonic point. If the sonic point is on the rounded corner of the body, the above Busemann solution applies if the body corner radius  $\, {\rm r}_{\rm c} \,$  is equated to the radius  $R_{b*}$  of a conic-section body at the same sonic point ordinate, y.

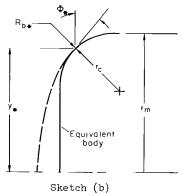
$$\frac{y_{*}}{R_{b}*} = \sin \Phi_{*}[(B_{b} - 1)\sin^{2}\Phi_{*} + 1]$$

$$= \frac{y_{*}}{r_{c}} = \frac{r_{m} - r_{c}(1 - \sin \Phi_{*})}{r_{c}}$$

or

$$\frac{r_c}{r_m} = \frac{1}{1 + (B_b - 1)\sin^3 \Phi_{\star}} \tag{3}$$

Equation (3) gives the  $\rm r_{\rm c}/\rm r_{\rm m}$  value for an equivalent body that supports the same elliptical shock and the same sonic angle as a body



with the bluntness parameter,  $B_b$ . The relationship between  $B_b$  and  $\Phi_\star$  was previously derived and is given by equation (2). The value of  $\sin\Phi_\star$  normalized with respect to the value for a sphere is plotted in figure 5 as a function of  $r_c/r_m$  as determined from equation (3). The general validity of this function at finite shock-density ratios will be discussed in a later section.

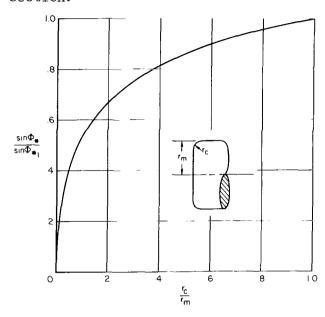


Figure 5.- Sonic angle,  $\Phi_*$ , on round-cornered bodies.

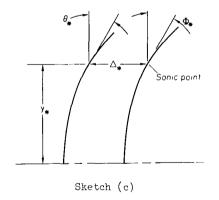


Figure 5 is intended to apply to any blunted body with a rounded corner. In many cases, however, a qualification of the sonic angle is necessary, for example when the body is conically or spherically blunted. The conical or spherical portion of the body will be tangent to the corner radius,  $r_c$ , at which point a surface inclination angle,  $\epsilon$ , is defined with respect to a plane normal to the body axis of symmetry. The sonic angle is then taken as  $\epsilon$  if  $\epsilon$  is greater than  $\Phi_*$  determined from figure 5.

Relationship between shock and body inclination. - Reference 2 shows that in a given free stream the shock angle,  $\theta_{\star}$ , opposite the sonic point on the body is uniquely related to the sonic point angle,  $\Phi_{\star}$ . The orientation of these angles is shown in sketch (c). The results of reference 2 are restricted to sharp-cornered bodies and are based on a gross mass-flow continuity analysis. It is nevertheless assumed that such a unique angle correspondence does exist and holds for all bodies with a detached shock. If this assumption is valid, the functional relationship between  $\theta_*$ and  $\Phi_{\star}$  can be established from any

set of shock solutions. The solutions of reference l for conic-section bodies were used for this purpose in the following manner: The value of  $y_*/R_s$  was found by solving equation (4) (appendix B); values of  $B_b$ ,  $B_s$ ,  $R_s/R_b$ , and  $\Delta_c/R_s$  were obtained from reference l and the value  $\Delta_*/y_*$  from figure 2:

$$\frac{\Delta_{*}y_{*}}{y_{*}R_{s}} = \frac{\Delta_{o}}{R_{s}} + \frac{R_{b}}{B_{b}R_{s}} \left[ 1 - \sqrt{1 - B_{b} \left( \frac{R_{s}y_{*}}{R_{b}R_{s}} \right)^{2}} \right] - \frac{1}{B_{s}} \left[ 1 - \sqrt{1 - B_{s} \left( \frac{y_{*}}{R_{s}} \right)^{2}} \right]$$
(4)

With the value of  $y_*/R_s$  determined, the following equations were then solved for  $\theta_*$  and  $\Phi_*$ :

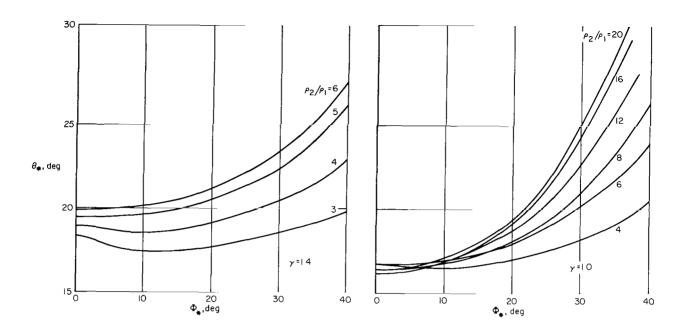
$$\tan \theta_{*} = \frac{\frac{y_{*}}{R_{s}}}{\sqrt{1 - B_{s} \left(\frac{y_{*}}{R_{s}}\right)^{2}}}$$
 (5)

$$\tan \Phi_{*} = \frac{\frac{y_{*}R_{s}}{R_{s}R_{b}}}{\sqrt{1 - B_{b}\left(\frac{y_{*}R_{s}}{R_{s}R_{b}}\right)^{2}}}$$
(6)

The above procedure, giving the relationship between  $\theta_{\star}$  and  $\Phi_{\star}$ , was restricted to the limited range of body bluntness  $(B_b < 3)$  (as found in ref. 1) and, therefore, to a limited range of  $\Phi_{\star}$ . Additional procedures were necessary to determine the value of  $\theta_{\star}$  at smaller values of  $\Phi_{\star}$  associated with bodies of larger bluntness. A simple solution for  $\theta_{\star}$  in the limiting case of a flat disk  $(B_b = \infty, \Phi_{\star} = 0^{\circ})$  was found (appendix C). The relationship between  $\theta_{\star}$  and  $\Phi_{\star}$  over the range of  $\Phi_{\star}$  not amenable to available theory was then approximated in the following manner.

It was assumed that if the corner of a flat disk was rounded to a small radius, no important change in the shock shape would occur. For the sonic point on the rounded corner the inclination  $\Phi_{\star}$  could be significantly different from zero. In the limit of a vanishingly small corner radius, no change occurs in the shock and, therefore,  $d\theta_{\star}=0$ . It then follows that  $d\theta_{\star}/d\Phi_{\star}=0$  at  $\Phi_{\star}=0^{\circ}$ , suggesting that  $\theta_{\star}$  would develop in an even power series in  $\Phi_{\star}$ . Accordingly, a three-term power series in  $\Phi_{\star}$  was used to approximate the variation in  $\theta_{\star}$  from its value at  $\Phi_{\star}=0^{\circ}$  to the values of  $\theta_{\star}$  (and  $d\theta_{\star}/d\Phi_{\star}$ ) in the range of  $\Phi_{\star}$  determined from the results of reference 1.

<sup>&</sup>lt;sup>2</sup>The small variation in shock shape with changes in body shape is pointed out in reference 1.

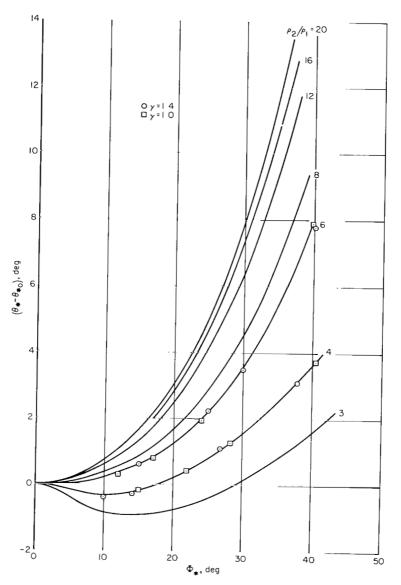


(a)  $\theta_{\star}$  as a function of  $\Phi_{\star}$ . Figure 6.- Inclination angles  $\theta_{\star}$  and  $\Phi_{\star}$ .

Plots of  $\theta_{\star}$  as a function of  $\Phi_{\star}$  are presented in figure 6(a) for various shock-density ratios and  $\gamma$  values of 1.4 and 1.0. A useful correlation between the results for the values of  $\gamma$  is presented in figure 6(b). The ordinate represents the incremental value or change in  $\theta_{\star}$  (defined as  $\theta_{\star}$  -  $\theta_{\star 0}$ ) as a function of  $\Phi_{\star}$ . At a given shock-density ratio, this incremental change in  $\theta_{\star}$  is independent of  $\gamma$ .

These relationships between  $\theta_*$  and  $\Phi_*$  do not completely agree with those of reference 2, primarily because of the restrictions imposed on the shock shape in the earlier work.

The manner in which the  $\theta_{*}$  -  $\Phi_{*}$  functions are applied in determining shock solutions for the classes of bodies considered here will be discussed in the following sections.

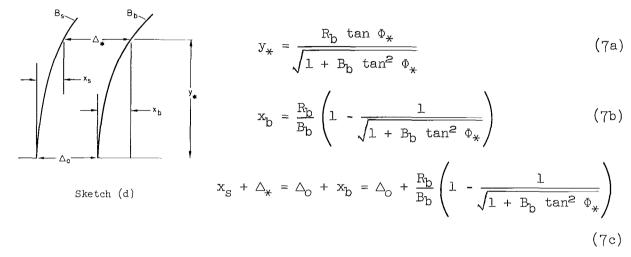


(b)  $(\theta_{*}$  -  $\theta_{*0})$  as a function of  $\Phi_{*}.$  Figure 6.- Concluded.

# Shock Envelope Solutions

Shock solutions will be considered for the following classes of bodies: ellipsoids ranging from a sphere to a flat disk, spherical or flat-faced bodies with an arbitrary degree of corner rounding, and spherically blunted, large-apex-angle cones. Equations relating the shock shape with the geometry of these bodies will be given. The use of a nomograph which simplifies the solution of these equations will be discussed.

Shock equations for conic-section body.- A conic-section body of known bluntness,  $B_b$ , and a shock bluntness,  $B_s$ , at a distance,  $\triangle_o$ , from the body are depicted in sketch (d). The geometrical relationships are:



Equation (7c) is divided by equation (7a) and the result is rearranged to give

$$\frac{\triangle_{o}}{y_{\star}} = \frac{\triangle_{\star}}{y_{\star}} - \frac{\sqrt{1 + B_{b} \tan^{2} \Phi_{\star}} - 1}{B_{b} \tan \Phi} + \frac{x_{s}}{y_{\star}}$$

or

$$\frac{R_{S}}{y_{*}} = \begin{bmatrix} \frac{\Delta_{*}}{y_{*}} - \frac{\sqrt{1 + B_{b} \tan^{2} \Phi_{*}} - 1}{B_{b} \tan^{\Phi}_{*}} \\ \frac{\Delta_{O}}{R_{S}} \end{bmatrix} + \frac{\frac{x_{S}}{y_{*}}}{\frac{\Delta_{O}}{R_{S}}}$$
(8)

Non-conic-section body shock equations. - A section of a body whose geometry<sup>3</sup> involves all the elements of the non-conic-section bodies to be considered is shown in sketch (e). The geometrical relationships are:

$$y_{*} = r_{m} - r_{c}(1 - \sin \Phi_{*}) \qquad (9a)$$

$$x_{b} = r_{m} \tan \epsilon + R_{b} \left(\frac{\cos \epsilon - 1}{\cos \epsilon}\right)$$

$$+ r_{c} \left(\frac{1 - \sin \epsilon}{\cos \epsilon} - \cos \Phi_{*}\right) \qquad (9b)$$

$$x_{s} + \Delta_{*} = \Delta_{o} + x_{b} = \Delta_{o} + r_{m} \tan \epsilon + R_{b} \left(\frac{\cos \epsilon - 1}{\cos \epsilon}\right)$$

$$+ r_{c} \left(\frac{1 - \sin \epsilon}{\cos \epsilon} - \cos \Phi_{*}\right) \qquad (9c)$$

Equation (9c) is divided by equation (9a) and the result is rearranged to give

$$\frac{\Delta_{\text{O}}}{y_{\text{*}}} = \frac{\Delta_{\text{*}}}{y_{\text{*}}} - \frac{r_{\text{m}} \tan \varepsilon + R_{\text{b}} \left( \frac{\cos \varepsilon - 1}{\cos \varepsilon} \right) + r_{\text{c}} \left( \frac{1 - \sin \varepsilon}{\cos \varepsilon} - \cos \Phi_{\text{*}} \right)}{r_{\text{m}} - r_{\text{c}} (1 - \sin \Phi_{\text{*}})} + \frac{x_{\text{s}}}{y_{\text{*}}}$$

or

$$\frac{R_{S}}{y_{*}} = \begin{bmatrix} \frac{\Delta_{*}}{y_{*}} - \tan \varepsilon + \frac{R_{b}}{r_{m}} \left( \frac{\cos \varepsilon - 1}{\cos \varepsilon} \right) + \frac{r_{c}}{r_{m}} \left( \frac{1 - \sin \varepsilon}{\cos \varepsilon} - \cos \Phi_{*} \right) \\ 1 - \frac{r_{c}}{r_{m}} \left( 1 - \sin \Phi_{*} \right) \\ \frac{\Delta_{o}}{R_{S}} \end{bmatrix} + \frac{\frac{x_{S}}{y_{*}}}{\frac{\Delta_{o}}{R_{S}}}$$

$$(10)$$

Shock solution nomograph. The equation of a shock, with the ordinate set equal to  $y_*$ , is

$$B_{S} \frac{X_{S}}{Y_{*}} = \left(\frac{R_{S}}{Y_{*}} - \frac{1}{\tan \theta_{*}}\right) \tag{11}$$

<sup>&</sup>lt;sup>3</sup>An important additional restriction of the method applies to spherically blunted, large-angle cones: The cone angle  $(90^{\circ} - \epsilon)$  must be sufficiently large to assure a detached shock even if the cone apex is sharp  $(R_h = 0)$ .

Since the body geometry is given,  $\Phi_{\star}$  can be determined from figure 3 or 5 and the corresponding value for  $\theta_{\star}$  can then be found from figure 6. Equations (1), (8) (or (10)), and (11) can be solved simultaneously to yield the shock solution in terms of  $y_{\star}/R_{\rm S}$ ,  $B_{\rm S}$ , and  $\Delta_{\rm O}/R_{\rm S}$ . This solution requires considerable computation. The computation can be minimized by use of a nomograph consisting of constant  $B_{\rm S}$  and  $\theta_{\star}$  curves on an  $R_{\rm S}/y_{\star}$  versus  $x_{\rm S}/y_{\star}$  coordinate system. Such curves, calculated with equation (11) and plotted in figure 7, are used as follows.

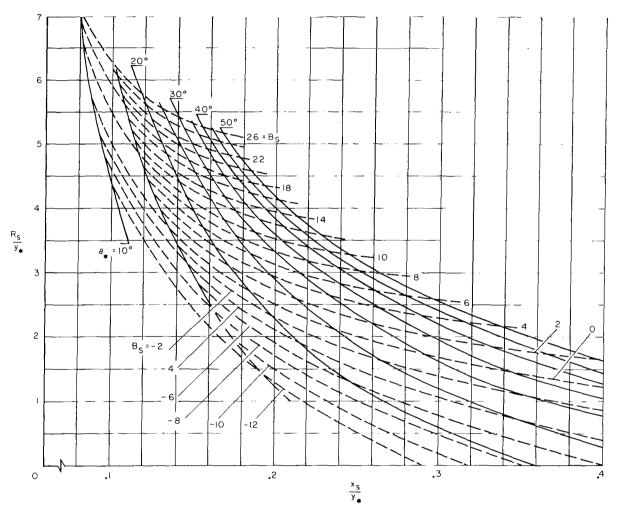


Figure 7.- Shock-solution nomograph.

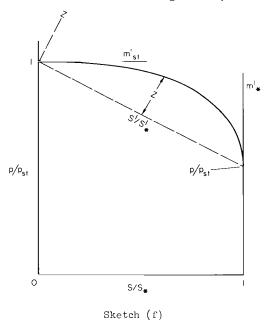
The numerator of the bracketed term in equation (8) or (10) contains elements of known value. If the value of  $\Delta_0/R_{\rm S}$  were also known, equation (8) or (10) could then be plotted as a straight line on the coordinate system. This straight line would have the ordinate value equal to the bracketed term at  $x_{\rm S}/y_{\star}=0$  and would have a slope equal to  $R_{\rm S}/\Delta_0$ . The shock solution would then be given by the intersection of the line with the nomograph curve having the appropriate known value of  $\theta_{\star}$ . This intersection point simultaneously locates the correct values for  $B_{\rm S}$  and  $R_{\rm S}/y_{\star}$ . The shock shape and its location with respect to the body are then completely defined.

In most cases,  $\triangle_{\rm O}/R_{\rm S}$  is not known so the procedure described above cannot be applied directly. However, a quick convergence to a solution is possible through an iterative technique beginning with a gross initial assumption for the value of  $\triangle_{\rm O}/R_{\rm S}$ . Details of this iterative technique will be clarified in the Numerical Examples section.

## Pressure Distribution

The method for estimating the pressure distribution on blunt bodies involves finding the simplest pressure distribution curve that conforms to "end" conditions specified at the stagnation and sonic points on the body. A suitable, easily determined curve was obtained through the use of the following coordinate system.

Coordinate system. - In the natural coordinate system of pressure and location points on the body, pressure can be expressed as some power series in terms of distance from the point of maximum (stagnation) pressure on the body. Experiments indicate that for a flat disk the pressure gradient is infinite at the sonic point (sketch (f)). For such a case, an infinite power



series in terms of distance from the sonic point is required to define the pressure distribution. In the present analysis, only a limited number of derivatives could be specified for the pressure distribution so that a term-by-term evaluation of an unlimited power series was not possible. However, if a pressure distribution curve without inflections could be represented in a coordinate system by a curve whose maximum gradient did not exceed a relatively small value, it would then be possible to approximate such a curve with a simple expression of few terms. dinate system in sketch (f) was constructed as follows: The origin of the coordinates was at  $s/s_* = 0$  and  $p/p_{st} = 1$ . The axis of the independent variable,  $s'/s_*'$ , was directed from this origin to the point  $p/p_{st} = p_{\star}/p_{st}$  at  $s/s_{\star} = 1$ . The transformed coordinates are indicated by dashed lines. The transformed vari-

ables are  $s'/s_*'$  and z. The absolute value of the slope of the transformed pressure distribution curve,  $z = f(s'/s_*')$ , cannot exceed the finite value,  $m_*' = 1/m_{st}'$ , as indicated. At  $s'/s_*' = 0$  and 1, z = 0.

The variation of z with  $s'/s'_*$  was approximated by two curves having appropriate slopes at  $s'/s'_*=0$  and  $s'/s'_*=1$ , respectively, and which were matched to a common tangent. The evaluation of these slopes, the resulting transformed pressure distribution functions, and the method of matching these functions to a common tangent are described in the following paragraphs.

Stagnation point. - The relationship between pressure and velocity for a perfect gas is

$$\frac{p}{p_{st}} = \left[1 - \frac{\gamma - 1}{\gamma + 1} \left(\frac{V}{V_*}\right)^2\right]^{\frac{\gamma}{\gamma - 1}}$$

which, for small velocities, V, near the stagnation point, can be written as

$$\frac{p}{p_{st}} \cong 1 - \frac{\gamma}{\gamma + 1} \left( \frac{V}{V_*} \right)^2 \tag{12}$$

The velocity variation with distance, s, is linear near the stagnation point so that equation (12) may be written, in natural coordinates, as

$$\frac{p}{p_{st}} = 1 - \left[ \nu \frac{s_*}{\Delta_0} \left( 1 + \frac{\Delta_0}{R_b} \right) \right]^2 \left( \frac{s}{s_*} \right)^2$$
 (13)

The bracketed term in equation (13) is discussed in detail in appendix D. The transformed function, z, in equation (14) is obtained from the Taylor series expansion in the neighborhood of the stagnation point where the second derivative is obtained from the radius of curvature found from equation (13):

$$z = m_{st}^{\dagger} \left( \frac{s^{\dagger}}{s_{\star}^{\dagger}} \right) - \left( 1 + m_{st}^{\dagger 2} \right)^{3/2} \left[ v \frac{s_{\star}}{\Delta_{o}} \left( 1 + \frac{\Delta_{o}}{R_{b}} \right) \right]^{2} \left( \frac{s^{\dagger}}{s_{\star}^{\dagger}} \right)^{2}$$

$$(14)$$

Equation (14) is assumed to give a close approximation to the value of z over a range of  $s'/s_*'$  to a matching point value that remains to be determined and depends upon the slope of z at the sonic point.

Sonic point. - Experimental pressure distribution data for flat-faced, round-cornered bodies indicate that the slope of the pressure distribution curve was inversely proportional to the radius of the body at the point where the pressure attained sonic value. The slope or derivative of the pressure distribution curve at the sonic point can be estimated quantitatively on the basis of an approximate theory discussed in appendix D. The result in natural coordinates is

$$\frac{d\left(\frac{p}{p_{st}}\right)}{d\left(\frac{s}{s_{\star}}\right)} \approx -\frac{s_{\star}}{r_{c}} \tag{15}$$

The slope of z at  $s'/s_*$  = 1 (sonic point) is

$$\frac{\mathrm{dz}}{\mathrm{d}\left(\frac{\mathrm{s}^{\dagger}}{\mathrm{s}^{\dagger}_{*}}\right)} = -\mathrm{tan}\left[\cot^{-1}\,\mathrm{m}^{\dagger}_{\mathrm{st}} - \cot^{-1}\left(\frac{\mathrm{s}_{*}}{\mathrm{r}_{\mathrm{c}}}\right)\right] = -\mathrm{m}^{\dagger}_{*} \tag{16}$$

in transformed coordinates.

Matching of curves. - Equation (14) is assumed to be valid over a range of  $s^{!}/s^{!}_{*}$  from zero to  $s^{!}_{t}/s^{!}_{*}$ . At  $s^{!}_{t}/s^{!}_{*}$  the curve defined by equation (14) is tangent to a second curve valid over the range  $s^{!}_{t}/s^{!}_{*} < s^{!}/s^{!}_{*} < 1$ . The equation of the second curve is assumed to have the same form as equation (14):

$$z = m_{\star}^{\bullet} \left( 1 - \frac{s^{\bullet}}{s_{\star}^{\bullet}} \right) - C \left( 1 - \frac{s^{\bullet}}{s_{\star}^{\bullet}} \right)^{2}$$
 (17)

The independent variable  $1 - (s'/s_*)$  is used for convenience.

Equations (14) and (17) are equated at  $s_t^{\bullet}/s_{\star}^{\bullet}$ . The derivative of equation (14) with respect to  $s_t^{\bullet}/s_{\star}^{\bullet}$  is equated to that of equation (17). Two independent equations are then made available to solve for the constant C in equation (17) and the point of tangency,  $s_t^{\bullet}/s_{\star}^{\bullet}$ . The results are

$$\frac{s_{t}^{i}}{s_{\star}^{i}} = \frac{m_{\star}^{i} - m_{st}^{i}}{m_{\star}^{i} + m_{st}^{i} - 2A}$$
 (18a)

where A is the absolute value of the coefficient of  $(s^{\, !}/s_{\, *}^{\, !})^2$  in equation (14) and

$$C = \frac{m_{st}^{!} + m_{*}^{!} - 2A \frac{s_{t}^{!}}{s_{*}^{!}}}{2 \left(1 - \frac{s_{t}^{!}}{s_{*}^{!}}\right)}$$
(18b)

The curves given by equations (14) and (17) are plotted over their respective ranges of applicability in the Z - (s'/s') coordinate system. To transform the results into the natural coordinate system,  $p/p_{st}$  versus  $s/s_*$ , it is only necessary to superpose the natural coordinates onto the z - (s'/s') curves. This is automatically accomplished with double coordinate paper (fig. 10).

The above procedure cannot predict pressure distributions if inflections in pressure occur. Such conditions can occur for a large-angle cone with the apex blunted by a small radius,  $R_{\text{b}}.$  However, this case is excluded and the method is applicable whenever  $s_{\text{t}}^{\prime}/s_{\text{x}}^{\prime}>0$  (eq. (18a)).

#### NUMERICAL EXAMPLES

#### Shock Solutions

The following numerical examples illustrate how the shape of the bow shock and its proximity to the body are determined for typical vehicles from each of the classes considered.

Example A. Ellipsoid: 
$$a/b = 0.25$$
,  $\gamma = 1.4$ ,  $M = 10$ 

At M = 10, the normal shock-density ratio for air is  $\rho_2/\rho_1$  = 5.71 or  $\rho_1/\rho_2$  = 0.175 thus G = 0.116 (fig. 1),  $\Delta_*/y_*$  = 0.325 (fig. 2),  $\sin \Phi_*/\sin \Phi_{*1}$  = 0.825 (fig. 3; a/b = 0.25),  $\Phi_*$  = 40.40 (fig. 4), and  $B_b$  = (b/a)<sup>2</sup> = 16 (by definition).

The values of  $\Phi_{\mathbf{x}}$  and  $\theta_{\mathbf{x}}$  are  $\Phi_{\mathbf{x}} = \sin^{-1}(0.825 \sin 40.4^{\circ}) = 32.3^{\circ}$  and,  $\theta_{\mathbf{x}} = 23.8^{\circ}$  (fig. 6(a)).

Equation (8) now reduces to

$$\frac{R_{s}}{y_{*}} = \begin{bmatrix} 0.325 - \frac{\sqrt{1 + 16(0.632)^{2}} - 1}{16(0.632)} + \frac{x_{s}}{\frac{\Delta_{o}}{R_{s}}} \\ \frac{\Delta_{o}}{R_{s}} \end{bmatrix}$$

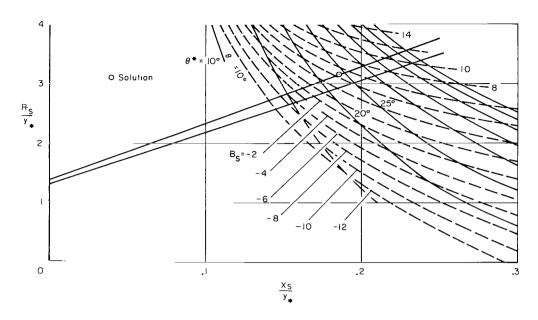
$$\frac{R_s}{y_*} = \frac{0.155}{\frac{\Delta_0}{R_s}} + \frac{\frac{x_s}{y_*}}{\frac{\Delta_0}{R_s}}$$

The value of  $\Delta_{\text{O}}/R_{\text{S}}$  is not known and cannot be determined with equation (1b) since the value of  $R_{\text{S}}/R_{\text{D}}$  is not known. If  $\Delta_{\text{O}}/R_{\text{S}}$  is approximated by  $\Delta_{\text{O}}/R_{\text{S}} \approx G$  = 0.116, the equation for  $R_{\text{S}}/y_{\star}$  becomes

$$\frac{R_s}{y_*} = \frac{0.155}{0.116} + \frac{1}{0.116} \frac{x_s}{y_*} = 1.336 + 8.621 \frac{x_s}{y_*}$$

The above equation is now plotted on the coordinates of figure 8(a). The equation is a straight line with the ordinate  $R_{\rm S}/y_{\star}=1.336$  at  $x_{\rm S}/y_{\star}=0$  and has a slope of 8.621. This line intersects the curve,  $\theta_{\star}=23.8^{\circ}$ , at  $R_{\rm S}/y_{\star}=2.98$ .

A trial value for  $R_{\rm s}/R_{\rm b}$  may now be determined from equation 7(a):



(a) Ellipsoid.

Figure 8. - Numerical application of nomograph.

$$\frac{y_{\star}}{R_{b}} = \frac{0.632}{\sqrt{1 + 16(0.632)^{2}}} = 0.232$$

and

$$\frac{R_s}{R_b} = \frac{R_s}{y_*} \frac{y_*}{R_b} = 2.98 \times 0.232 = 0.691$$

The value of  $\Delta_{o}/R_{s}$ , from equation (lb), is

$$\frac{\Delta_{0}}{R_{s}} = \frac{\sqrt{1 + 4(0.116)(0.691)} - 1}{2(0.691)} = 0.108$$

which is used to reevaluate  $R_s/y_*$ :

$$\frac{R_{s}}{y_{\star}} = \frac{0.155}{0.108} + \frac{1}{0.108} \frac{x_{s}}{y_{\star}} = 1.435 + 9.259 \frac{x_{s}}{y_{\star}}$$

and is found to give the value  $\rm R_s/y_*=3.15$  (fig. 8(a)) as the second trial solution. New values of  $\rm R_s/R_b$  and  $\rm \tilde{\Delta}_o/R_s$  are calculated:

$$\frac{R_s}{R_h}$$
 = 3.15 × 0.232 = 0.731

and

$$\frac{\triangle_0}{R_S} = \frac{\sqrt{1 + 4(0.116)(0.731)} - 1}{2(0.731)} = 0.108$$

No change in  $\Delta_{\rm O}/R_{\rm S}$  has occurred so convergence to a solution has been achieved. The value for  $B_{\rm S}$  is determined by the intersection point of the final equation for  $R_{\rm S}/y_{\star}$  with the curve,  $\theta$  = 23.80 (fig. 8(a)). The final results are:

$$B_{s} = 4.8$$

$$\frac{\Delta_{\rm O}}{R_{\rm S}} = 0.108$$

and

$$\frac{\Delta_{o}}{R_{b}} = 0.079$$

Example B. Flat-faced body with rounded corner:  $r_c/r_m = 0.25$ ,  $\gamma = 1.4$ , M = 5

At M = 5, the normal shock-density ratio for air is  $\rho_2/\rho_1$  = 5.00 or  $\rho_1/\rho_2$  = 0.200; thus G = 0.132 (fig. 1),  $\Delta_{\rm x}/y_{\rm x}$  = 0.372 (fig. 2),  $\sin \, \Phi_{\rm x}/\sin \, \Phi_{\rm x1}$  = 0.712 (fig. 5), and  $\Phi_{\rm x1}$  = 41.1° (fig. 4). The values  $\varepsilon$  = 0° and  $R_{\rm b}$  =  $\infty$  are appropriate to a flat-faced body.

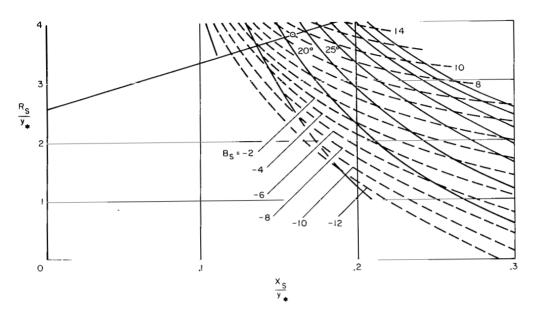
The values of  $\Phi_{\star}$  and  $\theta_{\star}$  are  $\Phi_{\star} = \sin^{-1}(0.712 \sin 41.1^{\circ}) = 27.9^{\circ}$  and  $\theta_{\star} = 21.8^{\circ}$  (fig. 6(a)).

Equation (10) is now evaluated. (Note that since  $R_b = \infty$ , equation (1) gives  $\Delta_0/R_s = G = 0.132$ .)

$$\frac{R_{s}}{y_{*}} = \frac{0.372 - \frac{0.0 + 0.0 + 0.250(1 - 0.884)}{1 - 0.250(1 - 0.468)} + \frac{x_{s}}{y_{*}}}{0.132}$$

$$\frac{R_s}{y_*}$$
 = 2.568 + 7.576  $\frac{x_s}{y_*}$ 

The line represented by the above equation has the  $\rm R_s/y_\star$  ordinate value of 2.568 at  $\rm x_s/y_\star=0$  and a slope of 7.576. This line intersects the curve,  $\theta_\star=21.8^{\rm O}$ , in figure 8(b) at  $\rm R_s/y_\star=3.77$  and also gives  $\rm B_s\approx 8.0$ . No iteration is necessary since  $\rm R_s/R_b=0$  remains fixed. The shock centerline radius and standoff distance may be related to the body radius,  $\rm r_m$ , with equation 9(a):



(b) Flat faced body.
Figure 8. - Continued.

$$\frac{y_*}{r_m} = 1 - 0.250(1 - 0.468) = 0.867$$

then

$$\frac{R_{\rm S}}{r_{\rm m}} = \frac{R_{\rm S}}{y_{\star}} \frac{y_{\star}}{r_{\rm m}} = 3.77 \times 0.867 = 3.27$$

and

$$\frac{\Delta_0}{r_m} = \frac{\Delta_0}{R_S} \frac{R_S}{r_m} = 0.132 \times 3.27 = 0.432$$

The final results are  $B_s = 8.0$ ,  $R_s/r_m = 3.27$ , and  $\Delta_o/r_m = 0.432$ .

Example C. Spherically blunted 65° cone with rounded corner:  $R_b/r_m = 0.167$ ,  $r_c/r_m = 0.083$ ,  $\gamma = 1.4$ , M = 5.2

At M = 5.2, the normal shock-density ratio of air is  $\rho_2/\rho_1$  = 5.064 or  $\rho_1/\rho_2$  = 0.1975; thus G = 0.1305 (fig. 1),  $\Delta_x/y_x$  = 0.368 (fig. 2),  $\sin \Phi_x/\sin \Phi_{x1}$  = 0.500 (fig. 5), and  $\Phi_{x1}$  = 41.05 (fig. 4). The value  $\epsilon$  = 90° - 65° = 25°.

The angle  $\Phi_* = \sin^{-1}(0.500 \sin 41.05^{\circ}) = 19.2^{\circ}$ . However, this value for  $\Phi_*$  is smaller than  $\epsilon = 25^{\circ}$ , the angle of tangency between the corner radius and the conical surface. Therefore, the sonic point is taken at the point of tangency:

$$\Phi_{\star} = \epsilon = 25^{\circ}$$

and

$$\theta_* = 21.4^{\circ} \text{ (fig. 6(a))}$$

Equation (10) reduces to

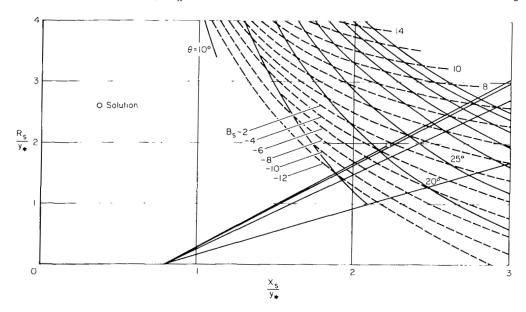
$$\frac{R_{s}}{y_{*}} = \frac{0.368 - \frac{0.466 + 0.167 \left(\frac{0.906 - 1}{0.906}\right) + 0.083 \left(\frac{1 - 0.423}{0.906} - 0.906\right)}{1 - 0.083(1 - 0.423)} + \frac{\frac{x_{s}}{y_{*}}}{\frac{\Delta_{o}}{R_{s}}}$$

$$\frac{R_{s}}{y_{\star}} = \frac{-0.080}{\frac{\Delta_{o}}{R_{s}}} + \frac{\frac{x_{s}}{y_{\star}}}{\frac{\Delta_{o}}{R_{s}}}$$

The value of  $\Delta_{\rm o}/R_{\rm s}$ , as in example A, is not known. If  $\Delta_{\rm o}/R_{\rm s}$  is approximated by G = 0.1305

$$\frac{R_{s}}{y_{*}} = \frac{-0.080}{0.1305} + \frac{1}{0.1305} \frac{x_{s}}{y_{*}} = -0.613 + 7.663 \frac{x_{s}}{y_{*}}$$

In this case, the line representing equation (10) crosses the abscissa axis of figure 8(c) at  $x_{\rm S}/y_{\star}$  = 0.080 (independent of the value for  $\triangle_{\rm O}/R_{\rm S}$ )



(c) Spherically blunted cone.
Figure 8.- Continued.

and has a slope of 7.663. The intersection of this line with the curve  $\theta_* = 21.4^{\circ}$  gives  $R_s/y_* = 1.34$ .

A trial value for  $R_s/R_b$  may now be determined. Equation (9a) gives

$$\frac{y_*}{r_m} = 1 - 0.083(1 - 0.423) = 0.9519$$

and

$$\frac{R_{s}}{R_{b}} = \frac{R_{s}}{y_{*}} \frac{y_{*}}{r_{m}} \frac{r_{m}}{R_{b}} = 1.34 \frac{0.9519}{0.167} = 1.34 \times 5.710 = 7.651$$

The value of  $\Delta_{\rm o}/R_{\rm s}$  , determined with equation (lb) using the above value for  $R_{\rm s}/R_{\rm b}$  , is

$$\frac{\Delta_0}{R_S} = \frac{\sqrt{1 + 4(0.1305)(7.651)} - 1}{2(7.651)} = 0.0807$$

Equation (10), reevaluated with the new value for  $\triangle_{O}/R_{S}$ , is

$$\frac{R_{s}}{y_{*}} = \frac{-0.080}{0.0807} + \frac{\frac{x_{s}}{y_{*}}}{0.0807} = -0.991 + 12.391 \frac{x_{s}}{y_{*}}$$

The intersection of the above line with the curve  $\theta_{\star}=21.4^{\circ}$  in figure 8(c) gives  $R_{\rm s}/y_{\star}=1.83$ . The adjusted values for  $R_{\rm s}/R_{\rm b}$  and  $\Delta_{\rm o}/R_{\rm s}$  are

$$\frac{R_s}{R_b}$$
 = 1.83 × 5.710 = 10.449

$$\frac{\Delta_{0}}{R_{s}} = \frac{\sqrt{1 + \frac{4(0.1305)(10.449)}{2(10.449)}} - 1}{2(10.449)} = 0.0737$$

then

$$\frac{R_s}{y_*} = \frac{-0.080}{0.0737} + \frac{\frac{x_s}{y_*}}{0.0737} = -1.085 + 13.569 \frac{x_s}{y_*}$$

The intersection of the above adjusted line with the  $\theta_*$  = 21.4° curve now gives  $R_{\rm S}/y_{\star}$  = 1.95. Another iteration is performed for  $R_{\rm S}/R_{\rm b}$  and  $\Delta_{\rm O}/R_{\rm S}$ :

$$\frac{R_s}{R_b}$$
 = 1.95 × 5.710 = 11.134

and

$$\frac{\triangle_{0}}{R_{s}} = \frac{\sqrt{1 + 4(0.1305)(11.134)} - 1}{2(11.134)} = 0.0722$$

Equation (10) is again adjusted to give

$$\frac{R_{s}}{y_{*}} = \frac{-0.080}{0.0722} + \frac{\frac{x_{s}}{y_{*}}}{0.0722} = -1.108 + 13.850 \frac{x_{s}}{y_{*}}$$

The value  $R_{\rm S}/y_{\star}=1.97$  is now determined from figure 8(c). No large change in  $R_{\rm S}/y_{\star}$  has occurred, indicating that convergence to a solution has been essentially accomplished. The value  $B_{\rm S}=-2.5$  is also determined from figure 8(c).

The final results are:

$$\begin{split} &B_{\rm S} = -2.5 \\ &\frac{R_{\rm S}}{R_{\rm b}} = 1.97 \times 5.710 = 11.249 \\ &\frac{\Delta_{\rm O}}{R_{\rm S}} = \frac{1 - 4(0.1305)(11.249)}{2(11.249)} - 1 = 0.0721 \\ &\frac{\Delta_{\rm O}}{r_{\rm m}} = \frac{\Delta_{\rm O}}{R_{\rm S}} \frac{R_{\rm S}}{y_{\star}} \frac{y_{\star}}{r_{\rm m}} = 0.0721 \times 1.97 \times 0.9519 = 0.135 \end{split}$$

Example D. Spherical-faced body with rounded corner:  $R_b/r_m$  = 2.4,  $r_c/r_m$  = 0.10, equilibrium flow with  $\rho_2/\rho_1$  = 20

The air is assumed to behave as a perfect gas, and since the Mach number is large, the asymptotic value,  $\gamma = [(\rho_2/\rho_1) + 1]/[(\rho_2/\rho_1) - 1] = 21/19 = 1.105$ , characterizes the thermodynamic properties of the air.

Since no body shock parameter charts are given here for gases with  $\gamma$  other than 1.0 and 1.4, interpolations are necessary.

The value G=0.037 is found from figure 1 at  $\rho_1/\rho_2=1/20=0.05$ . (The effect of  $\gamma$  on G is negligible at small values of  $\rho_1/\rho_2$ .) The values  $\Delta_{\chi}/y_{\chi}=0.117$  is found from figure 2 by linear interpolation between the curves for  $\gamma=1.0$  and  $\gamma=1.4$  at  $\rho_1/\rho_2=0.05$ . The value  $\sin \Phi_{\chi}/\sin \Phi_{\chi 1}=0.53$  is found from figure 5 at  $r_{\rm C}/r_{\rm m}=0.10$ . The value for  $\Phi_{\chi 1}$  is found from figure 4. Note that there is no great difference between the values of  $\Phi_{\chi 1}$  for gases with  $\gamma$  of 1.0 or 1.4 at the same shock-density ratio. The difference in  $\Phi_{\chi 1}$  between gases with  $\gamma=1.1$  and 1.0 should be negligible, particularly at small values of  $\rho_1/\rho_2$ . Accordingly, the value  $\Phi_{\chi 1}=36.8^{\circ}$  found from figure 4 at  $\rho_1/\rho_2=0.05$  and  $\gamma=1.0$  is assumed valid for  $\gamma=1.105$ .

The value of  $\,\varepsilon\,$  for a spherically blunted vehicle of radius  $\,R_b^{},\,$  tangent to a round corner of radius  $\,r_c^{},\,$  is given by

$$\epsilon = \sin^{-1} \left[ \frac{r_{m}}{R_{b}} \left( 1 - \frac{r_{c}}{r_{m}} \right) \right] = \sin^{-1} \left[ \frac{1}{2.4} \left( 1 - 0.10 \right) \right] = 23^{\circ}$$

The value  $\Phi_{\star} = \sin^{-1}(0.53 \sin 36.8^{\circ}) = 18.4^{\circ}$ . Since the angle  $\epsilon$  is larger than  $\Phi_{\star}$ ,  $\epsilon$  is taken as the sonic point inclination angle; therefore,  $\epsilon = \Phi_{\star} = 23^{\circ}$ .

The value  $\theta_{\star} - \theta_{\star 0} = 4.2^{\circ}$  is found from figure 6(b)  $(\rho_2/\rho_1 = 20 \text{ and } \Phi_{\star} = 23^{\circ})$ . The curves of figure 6(b) are independent of  $\gamma$ . The value of  $\theta_{\star 0}$ , given by equation (C4), is

$$\theta_{*0} = \tan^{-1} \frac{G}{\frac{\Delta_{*}}{y_{*}}} = \tan^{-1} \frac{0.037}{0.117} = 17.5^{\circ}$$

The value of  $\theta_{\star}$  is found by addition,  $(\theta_{\star} - \theta_{\star O}) + \theta_{\star O} = 17.5^{O} + 4.2^{O} = 21.7^{O}$ .

Equation (10) is now evaluated:

$$\frac{R_{s}}{y_{\star}} = \begin{bmatrix} 0.117 - & 0.424 + 2.4 & \frac{0.920 - 1}{0.920} + 0.10 & \frac{1 - 0.391}{0.920} - 0.920 \\ & & 1 - 0.10(1 - 0.391) \\ & & \frac{\Delta_{o}}{R_{s}} \end{bmatrix} + \frac{\frac{x_{s}}{y_{\star}}}{\frac{\Delta_{o}}{R_{s}}}$$

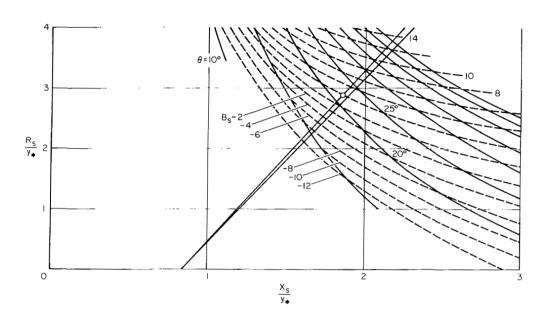
or

$$\frac{R_s}{y_*} = \frac{-0.084}{\Delta_0/R_s} + \frac{x_s/y_*}{\Delta_0/R_s}$$

If  $\triangle_{0}/R_{s}$  is approximated by G = 0.037, the equation for  $R_{s}/y_{*}$  becomes

$$\frac{R_s}{y_x} = \frac{-0.084}{0.037} + \frac{\frac{x_s}{y_x}}{0.037} = -2.270 + 27.03 \frac{x_s}{y_x}$$

In this case, as in example C, the line representing equation (10) crosses the  $x_{\rm s}/y_{\star}$  axis at  $x_{\rm s}/y_{\star}=0.08$ 4, as is apparent above. The slope of the line is 27.03. The intersection of the line with the  $\theta_{\star}=21.7^{\circ}$  curve in figure 8(d) gives the value  $R_{\rm s}/y_{\star}=2.82$ .



(d) Spherical-faced body. Figure 8. - Concluded.

Since the sonic point is at the tangent point of  $\rm R_b$  and  $\rm r_c$ ,  $\rm y_*=\rm R_b$  sin  $\rm \epsilon$ , or  $\rm y_*/\rm R_b=\sin 23^\circ=0.391$ . The value for  $\rm R_s/\rm R_b$  is then

$$\frac{R_s}{R_b} = \frac{R_s}{y_*} \frac{y_*}{R_b} = 2.82 \times 0.391 \equiv 1.103$$

The value of  $\triangle_0/R_s$ , determined with equation (1b), is

$$\frac{\triangle_0}{R_s} = \frac{\sqrt{1 + 4(0.037)(1.103)} - 1}{2(1.103)} = 0.0356$$

Equation (10) is reevaluated:

$$\frac{R_s}{y_*} = \frac{-0.084}{0.0356} + \frac{\frac{x_s}{y_*}}{0.0356} = -2.359 + 28.09 \frac{x_s}{y_*}$$

A new value of  $R_{\rm S}/y_{\star}$  = 2.93 is found from figure 8(d). Since the value of  $R_{\rm S}/y_{\star}$  has not changed greatly, convergence to a solution has been essentially achieved. The final values are

$$B_{\rm s} \approx 2$$

$$\frac{R_s}{R_h} = 2.93 \times 0.391 = 1.146$$

$$\frac{\triangle_0}{R_S} = 0.0356$$

and

$$\frac{\Delta_{0}}{r_{m}} = \frac{\Delta_{0}}{R_{s}} \frac{R_{s}}{y_{*}} \frac{y_{*}}{r_{m}} = 0.0356 \times 2.93 \times 0.9391 = 0.098$$

#### Pressure Distribution

An example calculation for the pressure distribution over the forward face of a blunt body is presented in the following paragraphs. Only one example is given since the calculative procedure, with obvious minor variations, is applicable to most bodies for which the shock solution has been determined.

The method is not applicable to certain cases of large-angle cones with the apex blunted by a small radius,  $R_{\rm b}$ . Criteria for applicability of the method are given in the Analysis section.

The example calculation applies to the round-cornered, flat-faced body at Mach number 5 for which the shock solution was calculated in figure 8(b). The pertinent shock and body parameters are  $r_{\rm c}/r_{\rm m}=$  0.25,  $\Phi_{\rm *}=27.9^{\circ}$ ,  $\Delta_{\rm o}/r_{\rm m}=$  0.432,  $\rho_{\rm 1}/\rho_{\rm 2}=$  0.20, and  $\gamma=$  1.4.

First, the numerical values for the coefficients of s'/s; and (s'/s;)^2 of equation (14) are determined. The values for  $m_{st}$ ,  $s_*/\Delta_0$ ,  $\Delta_0/R_b$ , and  $\nu$  are required. The quantity  $m_{st}$  represents the slope of the line drawn from  $p/p_{st}=1$  to  $p_*/p_{st}$  as indicated in sketch (f). In the present example, the gas is air for which  $p/p_{st}=0.528$ ; and

$$m_{st}^{\bullet} = 1 - \frac{p}{p_{st}} = 1 - 0.528 = 0.472$$

The length, s\*\*, extends from the stagnation point at the vehicle centerline to the sonic point at location  $\Phi_*$  on the rounded corner of radius  $r_c$ . For the present example, s\*\* has the value

$$s_{\star} = (r_{\rm m} - r_{\rm c}) + \left(\frac{\Phi_{\star} r_{\rm c}}{57 \cdot 3^{\circ}}\right)$$

which, expressed in terms of  $\triangle_0$ , is

$$\frac{s_{\star}}{\Delta_{0}} = \frac{r_{m}}{\Delta_{0}} \left( 1 - \frac{r_{c}}{r_{m}} \right) + \frac{\Phi_{\star}}{57 \cdot 3} \frac{r_{c}}{r_{m}} \frac{r_{m}}{\Delta_{0}}$$

or

$$\frac{s_*}{\Delta_0} = \frac{1}{0.432} (1 - 0.25) + \frac{27.9^{\circ}}{57.3^{\circ}} 0.25 \frac{1}{0.432} = 2.018$$

The value for  $\Delta_0/R_b$  is 0 since  $R_b=\infty$  for a flat face. The value  $\nu$  = 0.150 is found from figure 9 at  $\rho_1/\rho_2$  = 0.20 and  $\gamma$  = 1.4.

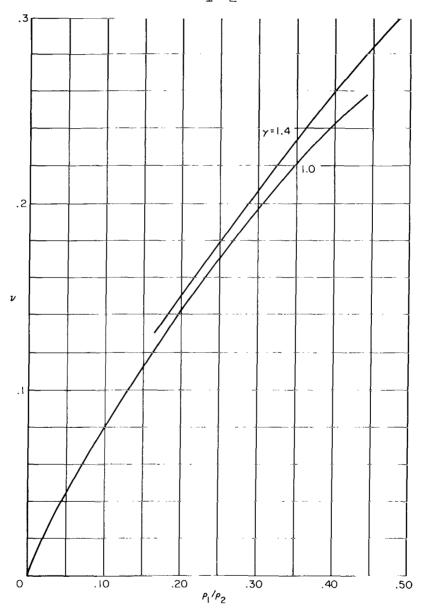


Figure 9. - The  $\nu$  function.

Equation (14), written with numerical coefficients, is

$$z = 0.472 \frac{s!}{s!} - (1 + 0.472^2)^{3/2} [0.150 \times 2.018(1 + 0.0)]^2 \left(\frac{s!}{s!}\right)^2$$

or

$$z = 0.472 \frac{s!}{s!} - 0.126 \left(\frac{s!}{s!}\right)^2$$

Next, the numerical values of the coefficients of 1 - s'/s; and  $[1 - (s'/s_*)]^2$  of equation (17) are determined. The values for  $s_*/r_c$ ,  $m_*$ ,  $s_t'/s_*$ , and C are required.

$$\frac{s_*}{r_0} = \frac{r_m}{r_0} \frac{\Delta_0}{r_m} \frac{s_*}{\Delta_0} = \frac{1}{0.25} 0.432 \times 2.018 = 3.487$$

Equation (16) is then evaluated for my

$$m_{*}^{!} = \tan \left( \cot^{-1} m_{st}^{!} - \cot^{-1} \frac{s_{*}}{r_{c}} \right) = \tan(\cot^{-1} 0.472 - \cot^{-1} 3.487)$$

$$= \tan \left( 64.7^{\circ} - 16.0^{\circ} \right) = 1.144$$

The value of A required to calculate the value of the coefficient C has already been determined and is the coefficient of  $(s'/s_*)^2$  in the equation for z, that is, A = 0.126. The value of the tangent point location  $s_t'/s_*$  is also required to evaluate C. Its value (given by eq. (18a)) is

$$\frac{s_{t}^{i}}{s_{x}^{i}} = \frac{1.144 - 0.472}{1.144 + 0.472 - 2 \times 0.126} = 0.4927$$

The coefficient C (given by eq. (18b)) may now be evaluated:

$$C = {0.472 + 1.144 - 2 \times 0.126 \times 0.4927 \over 2(1 - 0.4927)} = 1.471$$

Equation (17), written with numerical coefficients, is

$$z = 1.144 \left(1 - \frac{s!}{s!}\right) - 1.471 \left(1 - \frac{s!}{s!}\right)^{2}$$

Values for z in the range 0 < s  $^{1}/\mathrm{s}_{\star}^{\star}$  < 0.4927 computed with equation (14) are

s '/s*	Z
0.0	0.0
0.1	0.0459
0.2	0.0894
0.3	0.1303
0.4	0.1686
0.4927	0.2019

Values for z in the range 0.4927 < s  $^{\mbox{!}}/s_{\times}^{\mbox{!}}<$  1.0 computed with equation (17) are

s'/s;	_ Z
0.4927	0.2019
0.6	0.2222
0.7	0.2108
0.8	0.1697
0.9	0.0996
1.0	0.0

The above sets of values are shown plotted on the Z - (s'/s\*) coordinates of figure 10. The resulting curve represents the pressure distribution on the superposed (p/p\*st) - (s/s\*) coordinates.

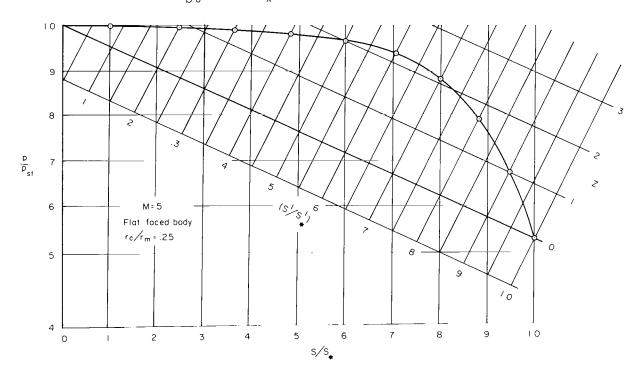


Figure 10.- The Z -  $(s'/s_*)$  coordinate system with an example pressure-distribution solution.

# COMPARISON OF EXPERIMENTAL AND PREDICTED RESULTS

The validity of the method was assessed on the basis of the experimental results of references 3, 4, 6, 7, and 8. These data are not extensive but do include representative cases among the classes of body shapes considered in the previous sections.

# Shock Shape

A comparison of predicted and experimental shock traces for various types of blunt bodies is shown in figure 11.

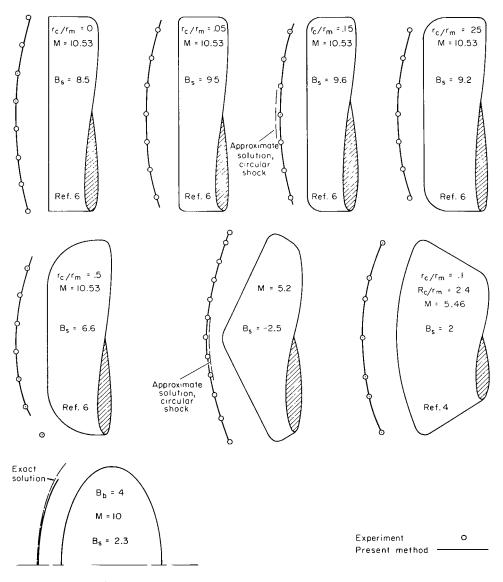


Figure 11. - Comparison of experimental and predicted shock shapes.

The geometric parameters of each body are indicated along with the bluntness,  $B_{\rm s}$ , of the associated shock and the Mach number of the free stream. Good agreement between experimental and predicted shock shapes is shown.

In all cases, the shock traces appear circular. However, the shock bluntness,  $B_{\rm S}$ , has a significant effect on the centerline shock standoff distance. To illustrate this effect, an element of a circular shock is shown on the body axis of symmetry for the cases,  $r_{\rm C}/r_{\rm m}=0.5$  and M=10.53, and for the conical body (example C), M=5.2. The circular shock satisfies the value,  $\Delta_{\rm x}/{\rm y}_{\rm x}$ , appropriate to the free-stream flow and the value,  $\Phi_{\rm x}$ , appropriate to the body geometry. An appreciable error is evident in the shock standoff distance at the centerline for the circular shock. This error would increase at larger values for the normal shock-density ratio.

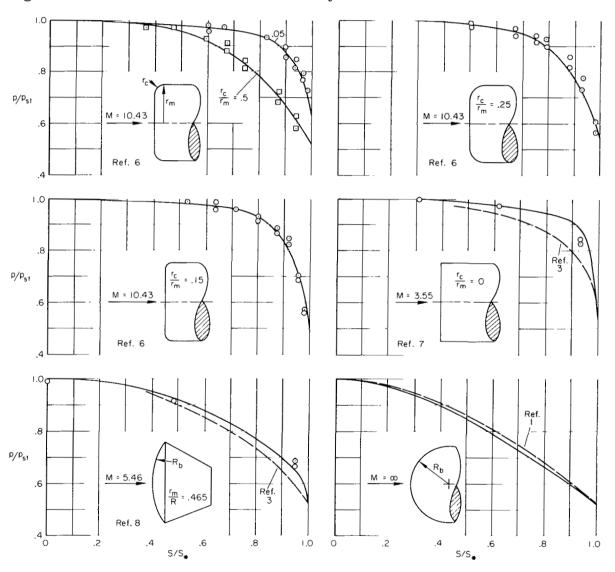


Figure 12.- Comparison of experimental and predicted pressure distributions.

The good agreement between predicted and experimental shock shapes verifies the correlation curve of  $\Delta_{\rm x}/{\rm y}_{\rm x}$  (fig. 2) and the value,  $\Phi_{\rm x}$ , determined from figure 5. The good agreement also confirms the relationship between  $\theta_{\rm x}$  and  $\Phi_{\rm x}$  (fig. 6) since the value of B<sub>s</sub>, which strongly influences the centerline shock standoff distance, is sensitive to the correct value for  $\theta_{\rm x}$ .

## Pressure Distribution

Predicted and experimental pressure distributions over the forward face of various blunt bodies are compared in figure 12. The results of the present method and the theoretical results of reference 1 for the sphere at  $M=\infty$  are also compared.

Agreement between the predicted and experimental results is usually within the scatter of the experimental values. The predicted values are somewhat higher than the experimental values for the sharp-cornered, flat-faced cylinder at M=3.55. Values predicted by the method of reference 3, which is applicable for the two sharp-cornered vehicles shown, are also indicated in figure 12; those predicted by the present method are in better accord with experiment.

#### CONCLUDING REMARKS

A method was developed for predicting shock envelopes and pressure distributions for a variety of blunt bodies at zero angle of attack. The method is restricted to those cases in which the bow shock is detached from the body and the flow over the forward face of the body is subsonic.

The method is based on correlation functions which relate the shock standoff distances at the stagnation and sonic points to the body geometry. These correlation functions were developed from the perfect gas solutions of reference 1 and depend primarily on the normal-shock density ratio modified to a small degree by the specific heat ratio of the gas. Since the effect of the specific-heat ratio is small, the present method should give adequate solutions for the equilibrium flows of real gases.

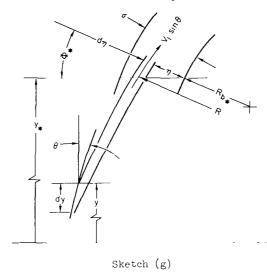
Predicted shock envelopes and pressure distributions were compared with experimental values for air flows in the Mach number range of 3.55 to 10.53 for a variety of body shapes. Satisfactory agreement between predicted and experimental values was found for both shock shapes and pressure distributions.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., 94035, Nov. 13, 1967
129-01-08-01-00-21

# APPENDIX A

## SONIC LOCATION ON ELLIPSOIDS

The sonic location on ellipsoids is found by the Busemann solution for infinite shock-density ratio.



The shock-body geometry in the vicinity of the sonic point of an ellipsoid is shown in sketch (g). The shock-layer thickness,  $\sigma$ , is measured normal to the body or in the direction inclined at the angle,  $\Phi_{\star}$ , with respect to the free-stream direction. The curve of radius, R, represents the streamline passing through the shock at a distance y from the axis of symmetry, at which point the shock inclination is  $\theta$ . The Busemann assumptions are made that the momentum tangent to the shock is preserved along streamlines within the shock layer, and that the velocity also remains constant along a streamline as indicated. In the limit of a vanishingly thin shock layer, the velocity

is  $V_1 \sin \theta = V_1 \sin \Phi$  and the stream radius, R, becomes equal to the local body radius,  $R_{hx}$ , at the sonic point.

A mass-flow continuity equation involving the differential annular flow tubes at the shock and within the shock layer, relating  $\,d\eta\,$  and  $\,y$ , is written

$$2\pi\rho_1V_1y dy = 2\pi\rho_2V_1y_* \sin \Phi d\eta$$

or

$$\frac{\rho_1 y \, dy}{\rho_2 y_* \sin \Phi} = d\eta \tag{Al}$$

The values of y and  $\Phi$  for an ellipsoid are related by

$$\frac{y}{R_b} = \frac{\sin \Phi}{\sqrt{(B_b - 1)\sin^2 \Phi + 1}} \tag{A2}$$

Equation (A2) is differentiated with respect to  $\Phi$  with the result

$$\frac{dy}{R_b} = \frac{\cos \Phi \, d\Phi}{\left[ (B_b - 1)\sin^2 \Phi + 1 \right]^{3/2}} \tag{A3}$$

The variable y is eliminated from equation (A1) with equations (A2) and (A3):

$$\frac{\rho_1 R_b^2}{\rho_2 y_*} \frac{\cos \Phi d\Phi}{\left[ (B_b - 1) \sin^2 \Phi + 1 \right]^2} = d\eta \tag{A4}$$

The differential pressure,  $\Delta p$ , across the shock layer at the sonic point  $(\Phi = \Phi_x)$  is given by

$$\Delta p = \int_{0}^{\sigma} \frac{dp}{d\eta} \ d\eta = \int_{0}^{\sigma} \frac{\rho_{2} V_{1}^{2} \sin^{2} \Phi \ d\eta}{R_{b*}} = \frac{\rho_{1} V_{1}^{2} R_{b}^{2}}{y_{*} R_{b*}} \int_{0}^{\Phi_{*}} \frac{\sin^{2} \Phi \cos \Phi \ d\Phi}{[(B_{b} - 1)\sin^{2} \Phi + 1]^{2}}$$

The radius,  $R_{b*}$ , of an ellipse at location  $\Phi_*$  is related to the centerline radius,  $R_{b}$ , by the identity

$$\frac{R_{b}^{2}}{y_{*}R_{b*}} = \frac{[(B_{b} - 1)\sin^{2}\Phi_{*} + 1]^{2}}{\sin\Phi_{*}}$$
 (A5)

The above integration was performed and, with equation (A5), the result was arranged to give the following equation for the differential pressure across the shock layer at the sonic point:

$$\frac{\Delta p}{\rho_1 V_1^2} = \frac{[(B_b - 1)\sin^2 \phi_* + 1]^2}{2(B_b - 1)\sin \phi_*} \left\{ \frac{1}{\sqrt{B_b - 1}} \sin^{-1} \left[ \frac{\sqrt{B_b - 1}\sin \phi_*}{\sqrt{(B_b - 1)\sin^2 \phi_* + 1}} \right] - \frac{\sin \phi_*}{(B_b - 1)\sin^2 \phi_* + 1} \right\}$$
(A6)

An independent equation giving this pressure differential for a gas with  $\gamma$  = 1 at infinite Mach number is the Newtonian result

$$\frac{\Delta p}{\rho_1 V_1^2} = (1 - \sin^2 \Phi_{\star}) - \frac{p_{\star}}{p_{st}} \tag{A7}$$

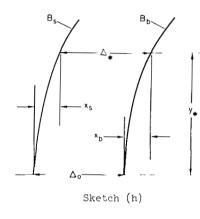
Equations (A6) and (A7) are equated to give

$$\frac{\left[\left(B_{b}-1\right)\sin^{2}\Phi_{*}+1\right]^{2}}{2\left(B_{b}-1\right)\sin\Phi_{*}} \left\{ \frac{1}{\sqrt{B_{b}-1}}\sin^{-1}\left[\frac{\sqrt{B_{b}-1}\sin\Phi_{*}}{\sqrt{\left(B_{b}-1\right)\sin^{2}\Phi_{*}+1}}\right] - \frac{\sin\Phi_{*}}{\left(B_{b}-1\right)\sin^{2}\Phi_{*}+1} \right\} \\
= \left(1-\sin^{2}\Phi_{*}\right) - \frac{p_{*}}{p_{st}} \tag{2}$$

## APPENDIX B

# THE $\theta_{\star}$ - $\Phi_{\star}$ FUNCTION

The relationship between  $\theta_*$  and  $\Phi_*$  was derived from the results from reference 1 for a sphere, along with the results found from the shock solutions of a series of increasingly (elliptically) blunted bodies. The solution for elliptically blunted bodies is developed in this section.



Sketch (h) depicts a body that is elliptically blunted,  $B_b$ , up to at least the sonic point at  $y_*$  and its associated shock of bluntness  $B_s$ .

By inspection,

$$\triangle_{x} = \triangle_{0} + x_{b} - x_{s}$$
 (B1)

The values of  $x_{\rm S}$  and  $x_{\rm b}$  are related to y through the bluntness parameters,  $B_{\rm S}$  and  $B_{\rm b}$ , as follows:

$$x_{s} = \frac{R_{s}}{B_{s}} \left[ 1 - \sqrt{1 - B_{s} \left( \frac{y_{*}}{R_{s}} \right)^{2}} \right]$$
(B2)

and

$$x_b = \frac{R_b}{B_b} \left[ 1 - \sqrt{1 - B_b \left( \frac{y_*}{R_b} \right)^2} \right]$$
 (B3)

The values of  $x_{\rm S}$  and  $x_{\rm b}$  given by equations (B2) and (B3) are substituted into equation (B1) and the result divided by  $R_{\rm S}$  to give

$$\frac{\Delta_{\star}}{y_{\star}} \frac{y_{\star}}{R_{s}} = \frac{\Delta_{o}}{R_{s}} + \frac{R_{b}}{B_{b}R_{s}} \left[ 1 - \sqrt{1 - B_{b} \left( \frac{R_{s}}{R_{b}} \frac{y_{\star}}{R_{s}} \right)^{2}} \right] - \frac{1}{B_{s}} \left[ 1 - \sqrt{1 - B_{s} \left( \frac{y_{\star}}{R_{s}} \right)^{2}} \right]$$

$$(4)$$

Equation (4) is then solved for  $y_*/R_s$ . The angles  $\theta_*$  and  $\Phi_*$  are then found from the following equations, which are derivatives of equations (B2) and (B3),

$$\tan \theta_{*} = \frac{\frac{y_{*}}{R_{s}}}{\sqrt{1 - B_{s} \left(\frac{y_{*}}{R_{s}}\right)^{2}}}$$
 (5)

$$\tan \Phi_{\star} = \frac{\frac{y_{\star}}{R_{s}} \frac{R_{s}}{R_{b}}}{\sqrt{1 - B_{b} \left(\frac{y_{\star}}{R_{s}} \frac{R_{s}}{R_{b}}\right)^{2}}}$$
(6)

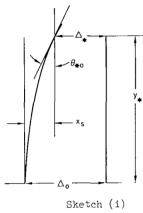
## APPENDIX C

# SHOCK BLUNTNESS $B_{\mathrm{S}}$ AND ANGLE $\theta_{\mathrm{*O}}$ FOR A DISK

On a flat disk the centerline body radius  $R_b = \infty$  and equation (1) gives

$$\frac{\triangle_{O}}{R_{S}} = G$$
 (C1)

The equation of the shock is written in the form



$$B_{S} \frac{x_{S}}{y_{*}} = \left(\frac{R_{S}}{y_{*}} - \frac{1}{\tan \theta_{*O}}\right) \tag{C2a}$$

where  $x_{\rm S}$  is the streamwise distance from the shock apex to the location on the shock at the distance  $\Delta_{\rm X}$  in the free-stream direction from the corner of the disk, as indicated in sketch (i). The angle  $\theta_{\rm XO}$  is the shock inclination; the length  $x_{\rm S}$  has the value  $\Delta_{\rm O}$  -  $\Delta_{\rm X}$ . Substituting this value for  $x_{\rm S}$  in equation (C2a) and with  $R_{\rm S} = \Delta_{\rm O}/G$  gives

$$B_{s}\left(\frac{\triangle_{o} - \triangle_{*}}{y_{*}}\right) = \left(\frac{\triangle_{o}}{y_{*}} \frac{1}{G} - \frac{1}{\tan \theta_{*0}}\right)$$
 (C2b)

which is written in the form

$$\frac{\Delta_{O}}{y_{\star}} \left( \frac{1}{G} - B_{S} \right) = \frac{\Delta_{\star}}{y_{\star}} \left( \frac{y_{\star}}{\Delta_{\star}} \frac{1}{\tan \theta_{\star O}} - B_{S} \right)$$
 (C2c)

A solution to equation (C2c) is given if

$$B_{S} = \frac{1}{G} \tag{C3}$$

and

$$\tan \theta_{*0} = \frac{G}{\frac{\Delta_0}{y_*}}$$
 (C4)

#### APPENDIX D

## PRESSURE DISTRIBUTION

The pressure distribution at the stagnation and sonic point locations of a body are developed as follows.

# Stagnation Point

In reference 2 it is shown that the stagnation point velocity gradient is

$$\frac{\mathrm{dV}}{\mathrm{ds}} = \frac{\mathrm{V_1}}{\Delta_{\mathrm{O}}} \left( 1 + \frac{\Delta_{\mathrm{O}}}{\mathrm{R_{\mathrm{b}}}} \right) \frac{\mathrm{f}}{2} \tag{Dla}$$

The function, f, of reference 2 is related to the function G as follows:

$$f = 2 \frac{\rho_1}{\rho_{st}} \left[ 1 - \left( \frac{\rho_2}{\rho_1} - 1 \right) G \right]$$
 (Dlb)

Thus,

$$\frac{\mathrm{d}V}{\mathrm{d}s} = \frac{V_1}{\Delta_0} \left( 1 + \frac{\Delta_0}{R_\mathrm{b}} \right) \frac{\rho_1}{\rho_{\mathrm{st}}} \left[ 1 - \left( \frac{\rho_2}{\rho_1} - 1 \right) G \right] \tag{Dlc}$$

If the velocity, V, is assumed to be linear with s in the vicinity of the stagnation point, equation (Dlc) may be integrated and normalized with respect to sonic ( ) $_{\star}$  values to yield

$$\frac{V}{V_{\star}} = \frac{V_{1}}{V_{\star}} \frac{s_{\star}}{\Delta_{O}} \left( 1 + \frac{\Delta_{O}}{R_{b}} \right) \frac{\rho_{1}}{\rho_{st}} \left[ 1 - \left( \frac{\rho_{2}}{\rho_{1}} - 1 \right) G \right] \frac{s}{s_{\star}}$$
(D2)

The relationship between pressure and velocity of a perfect gas, written in a form valid for small velocities, is

$$\frac{p}{p_{st}} = \left[1 - \frac{\gamma - 1}{\gamma + 1} \left(\frac{V}{V_{\star}}\right)^{2}\right]^{\frac{\gamma}{\gamma - 1}} \cong 1 - \frac{\gamma}{\gamma + 1} \left(\frac{V}{V_{\star}}\right)^{2} \tag{10}$$

Equations (D2) and (10) are combined to give

$$\frac{p}{p_{st}} = 1 - \left[ \nu \frac{s_{\star}}{\Delta_{o}} \left( 1 + \frac{\Delta_{o}}{R_{b}} \right) \right]^{2} \left( \frac{s}{s_{\star}} \right)^{2}$$
 (13)

where

$$v = \sqrt{\frac{\gamma}{\gamma + 1}} \frac{V_{1}}{V_{*}} \frac{\rho_{1}}{\rho_{st}} \left[ 1 - \left( \frac{\rho_{2}}{\rho_{1}} - 1 \right) G \right]$$

The function  $\nu$  was determined for gases with  $\gamma=1.0$  and 1.4 as a function of the normal shock-density ratio,  $\rho_1/\rho_2$  (fig. 10).

Sonic Point

Newtonian theory gives for pressure distribution,

$$\frac{p}{p_{st}} = \cos^2 \Phi \tag{D4}$$

The derivative of the pressure distribution with respect to  $\Phi$  is

$$\frac{d\left(\frac{p}{p_{st}}\right)}{d\Phi} = -2 \sin \Phi \cos \Phi \tag{D5}$$

The pressure gradient at the sonic point on a round corner of radius,  $r_c$ , is found by eliminating  $\Phi$  between equations (D4) and (D5):

$$\frac{d\left(\frac{p}{p_{st}}\right)}{ds} = \frac{d\left(\frac{p}{p_{st}}\right)}{r_{c} d\Phi} = -\frac{2}{r_{c}} \sqrt{\frac{p_{\star}}{p_{st}} \left(1 - \frac{p_{\star}}{p_{st}}\right)}$$

or, since, generally, 0.5  $< p_*/p_{st} < 0.6$ ,

$$\frac{d\left(\frac{p}{p_{st}}\right)}{d\left(\frac{s}{s_{*}}\right)} \approx -\frac{s_{*}}{r_{c}} \tag{15}$$

Although equation (15) was based on approximate theory, it generally agreed well with available experimental data.

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